The Ultimate ACT Guide to Imaginary Numbers

If you've been told your whole life that you can't take the square root of a negative number, your math teachers weren't lying—they were just keeping a secret. In the real number system, you can't. But on the ACT, you will encounter a special mathematical toolkit called **imaginary numbers.**

The good news? The ACT tests this concept in very predictable ways. Master these three core concepts, and you'll lock in those points.

1. The Core Definition: What is i?

The foundation of all imaginary numbers is the letter i. It is defined by one simple rule:

$$\mathbf{i} = \sqrt{-\mathbf{1}}$$

By extension, if you square both sides, you get the most important identity you need for the test:

$$\mathbf{i^2} = -\mathbf{1}$$

ACT Tip: Whenever you are simplifying an expression and end up with an i^2, immediately replace it with (-1).

Simplifying Radicals with Negative Numbers

If you see a negative inside a square root, just pull it out as an i:

- $\sqrt{-9} = \sqrt{9} \cdot \sqrt{-1} = 3i$
- $\sqrt{-50} = \sqrt{25} \cdot \sqrt{2} \cdot \sqrt{-1} = 5i\sqrt{2}$

2. The Cyclical Pattern of i

The ACT loves to ask you to simplify i raised to a high exponent, like i^{42} or i^{103}. To solve these, you just need to know that i repeats in a **4-step cycle**:

Power of i	Simplified Value	How to remember it
i^1	i	Just itself
i^2	-1	The core definition
i^3	$-i$	$i^2 \cdot i = -1 \cdot i$
i^4	1	$i^2 \cdot i^2 = -1 \cdot -1$

The ACT Shortcut for High Powers:

To find the value of i^n for any large number n:

1. **Divide the exponent by 4.**
2. **Look only at the remainder.** 3. Match the remainder to the cycle:
   - Remainder **.25** (or 1) $\rightarrow i$
   - Remainder **.50** (or 2) $\rightarrow -1$
   - Remainder **.75** (or 3) $\rightarrow -i$
   - Remainder **.00** (no remainder) $\rightarrow 1$

Example: What is i^{26}?
$26 \div 4 = 6.5$. Because the decimal is $.50$ (halfway, meaning a remainder of 2), $i^{26} = i^2 = -1$.

3. Operations with Complex Numbers

A **complex number** is just a mix of a real number and an imaginary number, written in the standard form:

$$\mathbf{a + bi}$$

On the ACT, you will be asked to add, subtract, and multiply these. Treat i just like the variable x, with one exception: **never leave an i^2 in your final answer.**

Adding & Subtracting (Combine Like Terms)

Just add the real parts together and the imaginary parts together.

$$(3 + 5i) + (2 - 7i) = (3 + 2) + (5i - 7i) = 5 - 2i$$

Multiplying (Use FOIL)

When multiplying two complex binomials, use the FOIL method (First, Outer, Inner, Last), then simplify i^2.

Example: Multiply $(2 + 3i)(1 - 4i)$

1. **FOIL it:** * First: $2 \cdot 1 = 2$
   - Outer: $2 \cdot (-4i) = -8i$
   - Inner: $3i \cdot 1 = 3i$
   - Last: $3i \cdot (-4i) = -12i^2$
   - Combined: $2 - 5i - 12i^2$
2. **Replace i^2 with -1:**

$$2 - 5i - 12(-1)$$

$$2 - 5i + 12$$

3. **Combine like terms:**

$$14 - 5i$$

Quick ACT Checklist

- [ ] Did you see a negative under a square root? Pull it out as i.
- [ ] Do you have an i^2 in your work? Change it to -1 right away.
- [ ] Is there a massive exponent on i? Divide by 4 and check the remainder.
- [ ] Are you multiplying? FOIL first, then combine the middle terms and fix the i^2 term.